Series

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1 Types of Series

1.1 Geometric Series

$\sum_{n=1}^{\infty} ar^n$ Converges if $|r| < 1$
If it converges, you can find the sum by $\frac{a}{1-r}$
Here r is the common ratio and a is the first term. You can find the common ratio by $\frac{a_{n+1}}{a_n}$

1.2 P-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

Converges if $p > 1$, diverges if $p \leq 1$

1.3 Telescoping Series

$$\sum_{n=1}^{\infty} (\frac{1}{n} - \frac{1}{n+1})$$

1.4 Harmonic Series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

1.5 Alternating Series

$$\sum_{n=1}^{\infty} (-1)^n a_n$$

2 Tests for Convergence

2.1 Integral Test

if $f(n) = a_n$, is positive, continuous, and decreasing, we can integrate with an improper integral. If the integral diverges, the series diverges.

2.2 Comparison Test

If $0 \leq a_n \leq b_n$
- if b_n converges, then a_n converges.
- if b_n diverges, then a_n diverges

2.3 Limit Comparison Test

$\lim_{x \to +\infty} \frac{a_n}{b_n} = C$
Where C is a finite number and positive, then both series converge or diverge.

2.4 Alternating Series Test

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$$

If series:
a. $b_{n+1} \leq b_n, \forall n$
b. $\lim_{n \to +\infty} b_n = 0$
Then the series converges.

2.5 The Test for Divergence

If $\lim_{n \to +\infty} a_n \neq 0$ then series a_n is divergence.

2.6 Root Test

$\lim_{n \to +\infty} \sqrt[n]{a}$
$L > 1$: Divergent
$L < 1$: Absolutely Convergent $L = 1$: Inconclusive

2.7 Ratio Test

$\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} \neq L$
$L > 1$: Divergent
$L < 1$: Absolutely Convergent
$L = 1$: Inconclusive